

UNIT – I : RELATIONS AND FUNCTIONS

CHAPTER

1

RELATIONS AND FUNCTIONS



Syllabus

Types of relations : Reflexive, Symmetric, Transitive and Equivalence relations. One-to-one and onto functions.

In this chapter you will study

- Different types of relations- Reflexive, Symmetric, Transitive and Equivalence relations.
- Different types of functions - Injective, Surjective and Bijective functions.

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Topic-1

Relations

Concepts Covered • Types of relations and their identification • Equivalence class



Revision Notes

1. Definition

A relation R , from a non-empty set A to another non-empty set B is mathematically as an subset of $A \times B$. Equivalently, any subset of $A \times B$ is a relation from A to B .

Thus, R is a relation from A to B

$$\Leftrightarrow R \subseteq A \times B$$

$$\Leftrightarrow R \subseteq \{(a, b) : a \in A, b \in B\}$$

Illustrations:

- (a) Let $A = \{1, 2, 4\}$, $B = \{4, 6\}$. Let $R = \{(1, 4), (1, 6), (2, 4), (2, 6), (4, 4), (4, 6)\}$. Here $R \subseteq A \times B$ and therefore R is a relation from A to B .
- (b) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 7\}$, Let $R = \{(2, 3), (3, 5), (5, 7)\}$. Here $R \not\subseteq A \times B$ and therefore R is not a relation from A to B . Since $(5, 7) \in R$ but $(5, 7) \notin A \times B$.
- (c) Let $A = \{-1, 1, 2\}$, $B = \{1, 4, 9, 10\}$ let $a \in A$ and $b \in B$ and $a R b$ means $a^2 = b$ then, $R = \{(-1, 1), (1, 1), (2, 4)\}$.

Note:

- A relation from A to B is also called a relation from A into B .
- $(a, b) \in R$ is also written as aRb (read as a is related to b).
- Let A and B be two non-empty finite sets having p and q elements respectively. Then $n(A \times B) = n(A) \cdot n(B) = pq$. Then total number of subsets of $A \times B = 2^{pq}$. Since each subset of $A \times B$ is a relation from A to B , therefore total number of relations from A to B will be 2^{pq} .

2. Domain & range of a relation

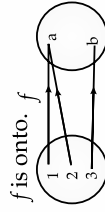
- (a) **Domain of a relation:** Let R be a relation from A to B . The domain of relation R is the set of all those elements $a \in A$ such that $(a, b) \in R \forall b \in B$.

Thus, $\text{Dom.}(R) = \{a \in A : (a, b) \in R \forall b \in B\}$.

$f : X \rightarrow Y$ is one-one if
 $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
 $\forall x_1, x_2 \in X$. Otherwise,
 f is many-one,
 f is one-one.



$f : X \rightarrow Y$ is onto if every
 $y \in Y, \exists x \in X$ such that $f(x) = y$.
 Then f is surjective

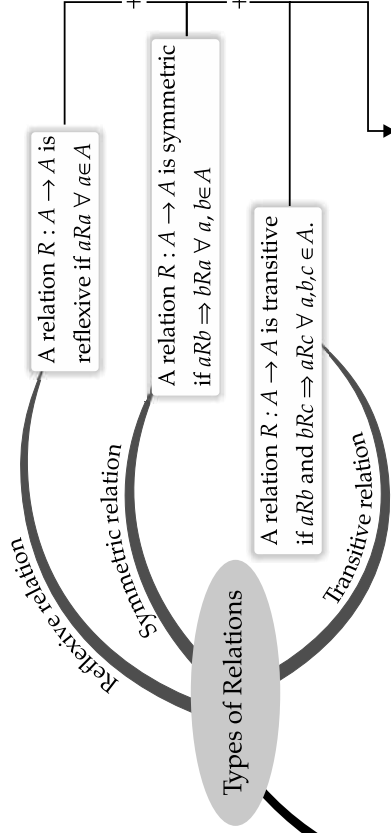
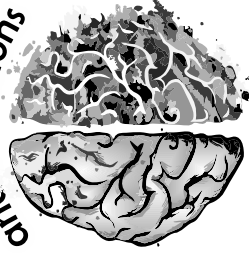


One-one (injective)

Types of Functions

(Surjective) Onto

Relations
and Functions



A relation $R : A \rightarrow A$ is reflexive if $aRa \forall a \in A$

A relation $R : A \rightarrow A$ is symmetric if $aRb \Rightarrow bRa \forall a, b \in A$

A relation $R : A \rightarrow A$ is transitive if aRb and $bRc \Rightarrow aRc \forall a, b, c \in A$.

Equivalence relation

(If a relation has reflexive, symmetric and transitive relations) e.g., Let $T =$ the set of all triangles in a plane and $R : T \rightarrow T$ defined by $R = \{(T_1, T_2)\} : T_1$ is congruent to $T_2\}$. Then, R is equivalence.

Trace the Mind Map

► First Level ► Second Level ► Third Level

That is, the domain of R is the set of first components of all the ordered pairs which belong to R .

- (b) **Range of a relation:** Let R be a relation from A to B . The range of relation R is the set of all those elements $b \in B$ such that $(a, b) \in R \forall a \in A$.

Thus, Range of $R = \{b \in B : (a, b) \in R \forall a \in A\}$.

That is, the range of R is the set of second components of all the ordered pairs which belong to R .

- (c) **Co-domain of a relation:** Let R be a relation from A to B . Then B is called the co-domain of the relation R . So we can observe that co-domain of a relation R from A into B is the set B as a whole.

Illustrations: Let $a \in A$ and $b \in B$ and

- (i) Let $A = \{1, 2, 3, 7\}, B = \{3, 6\}$. If aRb means $a < b$.

Then we have

$$R = \{(1, 3), (1, 6), (2, 3), (2, 6), (3, 6)\}.$$

Here, $\text{Dom}(R) = \{1, 2, 3\}$,

Range of $R = \{3, 6\}$, Co-domain of $R = B = \{3, 6\}$

- (ii) Let $A = \{1, 2, 3\}, B = \{2, 4, 6, 8\}$.

If $R_1 = \{(1, 2), (2, 4), (3, 6)\}$,

and $R_2 = \{(2, 4), (2, 6), (3, 8), (1, 6)\}$

Then both R_1 and R_2 are related from A to B because

$$R_1 \subseteq A \times B, R_2 \subseteq A \times B$$

Here, Dom

$$(R_1) = \{1, 2, 3\}, \text{Range of } R_1 = \{2, 4, 6\};$$

$$\text{Dom}(R_2) = \{2, 3, 1\}, \text{Range of } R_2 = \{4, 6, 8\}$$

3. Types of relations from one set to another set

- (a) **Empty relation:** A relation R from A to B is called an empty relation or a void relation from A to B if $R = \phi$.

For example, Let

$$A = \{2, 4, 6\}, B = \{7, 11\}$$

Let $R = \{(a, b) : a \in A, b \in B \text{ and } |a - b| \text{ is even}\}$.

Here R is an empty relation.

- (b) **Universal relation:** A relation R from A to B is said to be the universal relation if $R = A \times B$.

For example, Let

$$A = \{1, 2\}, B = \{1, 3\}$$

Let $R = \{(1, 1), (1, 3), (2, 1), (2, 3)\}$.

Here, $R = A \times B$, so relation R is a universal relation.

Note:

- The void relation i.e., ϕ and universal relation i.e., $A \times A$ on A are respectively the smallest and largest relations defined on the set A . Also these are also called **Trivial Relations** and other relation is called a **Non-Trivial Relation**.
- The relations $R = \phi$ and $R = A \times A$ are two **extreme relations**.

- (c) **Identity relation:** A relation R defined on a set A is said to be the identity relation on A if $R = \{(a, b) : a \in A, b \in A \text{ and } a = b\}$
Thus identity relation $R = \{(a, a) : \forall a \in A\}$

The identity relation on set A is also denoted by I_A .

For example, Let $A = \{1, 2, 3, 4\}$,

Then $I_A = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$.

But the relation given by

$$R = \{(1, 1), (2, 2), (1, 3), (4, 4)\}$$

is not an identity relation because element of I_A is not related to elements 1 and 3.

Note:

- In an identity relation on A every element of A should be related to itself only.

- (d) **Reflexive relation:** A relation R defined on a set A is said to be reflexive if $a R a \forall a \in A$ i.e., $(a, a) \in R \forall a \in A$.

For example, Let $A = \{1, 2, 3\}$ and R_1, R_2, R_3 be the relations given as

$$R_1 = \{(1, 1), (2, 2), (3, 3)\},$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3)\} \text{ and}$$

$$R_3 = \{(2, 2), (2, 3), (3, 2), (1, 1)\}$$

Here R_1 and R_2 are reflexive relations on A but R_3 is not reflexive as $3 \in A$ but $(3, 3) \notin R_3$.

Note:

- The universal relation on a non-void set A is reflexive.
- The identity relation is always a reflexive relation but the converse may or may not be true. As shown in the example above, R_1 is both identity as well as reflexive relation on A but R_2 is only reflexive relation on A .

- (e) **Symmetric relation:** A relation R defined on a set A is symmetric if

$$(a, b) \in R \Rightarrow (b, a) \in R \forall a, b \in A \text{ i.e., } aRb \Rightarrow bRa \text{ (i.e., whenever } aRb \text{ then } bRa).$$

For example, Let $A = \{1, 2, 3\}$,

$$R_1 = \{(1, 2), (2, 1)\}, R_2 = \{(1, 2), (2, 1), (1, 3), (3, 1)\}.$$

$$R_3 = \{(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)\}$$

$$R_4 = \{(1, 3), (3, 1), (2, 3)\}$$

Here R_1, R_2 and R_3 are symmetric relations on A .

But R_4 is not symmetric because $(2, 3) \in R_4$ but $(3, 2) \notin R_4$.

- (f) **Transitive relation:** A relation R on a set A is transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ i.e., aRb and $bRc \Rightarrow aRc$.

For example, Let $A = \{1, 2, 3\}$,

$$R_1 = \{(1, 2), (2, 3), (1, 3), (3, 2)\}$$

$$\text{and } R_2 = \{(1, 3), (3, 2), (1, 2)\}$$

Here R_2 is transitive relation whereas R_1 is not transitive because $(2, 3) \in R_1$ and $(3, 2) \in R_1$ but $(2, 2) \notin R_1$.

- (g) **Equivalence relation:** Let A be a non-empty set, then a relation R on A is said to be an equivalence relation if

- (i) R is reflexive i.e.,

$$(a, a) \in R \forall a \in A \text{ i.e., } aRa.$$

(ii) R is symmetric i.e.,

$$(a, b) \in R$$

$$\Rightarrow (b, a) \in R \quad \forall a, b \in A \text{ i.e., } aRb \Rightarrow bRa.$$

(iii) R is transitive i.e.,

$$(a, b) \in R \text{ and } (b, c) \in R$$

$$\Rightarrow (a, c) \in R \quad \forall a, b, c \in A$$

$$\text{i.e., } aRb \text{ and } bRc \Rightarrow aRc.$$

For example, Let $A = \{1, 2, 3\}$

$$R = \{(1, 2), (1, 1), (2, 1), (2, 2), (3, 3), (1, 3), (3, 1), (3, 2), (2, 3)\}$$

Here R is reflexive, symmetric and transitive. So R is an equivalence relation on A .

Equivalence classes: Let A be an equivalence relation in a set A and let $a \in A$. Then, the set of all those elements of A which are related to a , is called equivalence class determined by a and is denoted by $[a]$. Thus, $[a] = \{b \in A : (a, b) \in A\}$



Mnemonics

Types of relation

RIPE STRAWBERRY TO EAT

Interpretations

Ripe – reflexive

Strawberry – Symmetric

To – transitive

Eat - Equivalence

Note:

- Two equivalence classes are either disjoint or identical.
- An equivalence relation R on a set A partitions the set into mutually disjoint equivalence classes.
- An important property of an equivalence relation is that it divides the set into pair-wise disjoint subsets called **equivalence classes** whose collection is called a **partition of the set**.

Note that the union of all equivalence classes give the whole set.

e.g., Let R denotes the equivalence relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Then the equivalence class $[0]$ is $[0] = [0, \pm 2, \pm 4, \pm 6, \dots]$.



Key Words

Disjoint: These are sets which have no elements in common.

4. Tabular representation of a relation

In this form of representation of a relation R from set A to set B , elements of A and B are written in the first column and first row respectively. If $(a, b) \in R$ then we write '1' in the row containing a and column containing b and if $(a, b) \notin R$ then we write '0' in the same manner.

For example, Let $A = \{1, 2, 3\}$,

$B = \{2, 5\}$ and $R = \{(1, 2), (2, 5), (3, 2)\}$, then

R	2	5
1	1	0
2	0	1
3	1	0

5. Inverse relation

Let $R \subseteq A \times B$ be a relation from A to B . Then, the inverse relation of R , to be denoted by R^{-1} , is a relation from B to A defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

Thus $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1} \quad \forall a \in A, b \in B$.

Clearly, **Domain** (R^{-1}) = **Range** of R , **Range** of R^{-1} = **Domain** (R).



Key Words

Domain and Range: The set of x coordinate values is called domain and the set of y coordinate values is called range.

Also, $(R^{-1})^{-1} = R$.

For example, Let $A = \{1, 2, 4\}$, $B = \{3, 0\}$ and let $R = \{(1, 3), (4, 0), (2, 3)\}$ be a relation from A to B , then $R^{-1} = \{(3, 1), (0, 4), (3, 2)\}$.



Key Facts

- (i) A relation R from A to B is an empty relation or void relation if $R = \phi$
(ii) A relation R on a set A is an empty relation or void relation if $R = \phi$
- (i) A relation R from A to B is a universal relation if $R = A \times B$.
(ii) A relation R on a set A is an universal relation if $R = A \times A$.
- A relation R on a set A is reflexive if $aRa, \forall a \in A$.
- A relation R on a set A is symmetric if whenever aRb , then bRa for all $a, b \in A$.
- A relation R on a set A is transitive if whenever aRb and bRc then aRc for all $a, b, c \in A$.
- A relation R on A is identity relation if $R = \{(a, a) \mid a \in A\}$ i.e., R contains only elements of the type $(a, a) \forall a \in A$ and it contains no other element.

7. A relation R on a non-empty set A is an equivalence relation if the following conditions are satisfied :

- (i) R is **reflexive** i.e., for every $a \in A$, $(a, a) \in R$ i.e., aRa .
- (ii) R is **symmetric** i.e., for $a, b \in A$, $aRb \Rightarrow bRa$ i.e., $(a, b) \in R \Rightarrow (b, a) \in R$.
- (iii) R is **transitive** i.e., for all $a, b, c \in A$, we have, aRb and $bRc \Rightarrow aRc$ i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.



TYPES OF INTERVALS

- (i) **Open Intervals:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying strictly between a and b is called an open interval. It is denoted by $]a, b[$ or (a, b) i.e., $\{x \in R : a < x < b\}$.
- (ii) **Closed Intervals:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it includes both a and b as well is known as a closed interval. It is denoted by $[a, b]$ i.e., $\{x \in R : a \leq x \leq b\}$.
- (iii) **Open Closed Interval:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it excludes a and includes only b is known as an open closed interval. It is denoted by $]a, b]$ or $(a, b]$ i.e., $\{x \in R : a < x \leq b\}$.
- (iv) **Closed Open Interval:** If a and b be two real numbers such that $a < b$ then, the set of all the real numbers lying between a and b such that it includes only a and excludes b is known as a closed open interval. It is denoted by $[a, b[$ or $[a, b)$ i.e., $\{x \in R : a \leq x < b\}$.

Example 1

Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b) R(c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

Sol.

Step I : Given $(a, b) R(c, d)$ as $ad(b + c) = bc(a + d)$

$$\therefore \forall a, b \in N$$

$$\text{or } ab(b + a) = ba(a + b)$$

$$\text{or } (a, b) R(a, b)$$

$$\therefore R \text{ is reflexive.} \quad \dots(i)$$

Step II : Let $(a, b) R(c, d)$ for $(a, b), (c, d) \in N \times N$

$$\therefore ad(b + c) = bc(a + d) \quad \dots(ii)$$

$$\text{Also, } (c, d) R(a, b)$$

$$\therefore cb(d + a) = da(c + b)$$

[By commutation of addition and multiplication on N]

$$\therefore R \text{ is symmetric.} \quad \dots(iii)$$

Step III : Let $(a, b) R(c, d)$ and $(c, d) R(e, f)$ for $a, b, c, d, e, f \in N$

$$\therefore ad(b + c) = bc(a + d) \quad \dots(iv)$$

$$\text{and } cf(d + e) = de(c + f) \quad \dots(v)$$

Dividing eqn. (iv) by $abcd$ and eqn. (v) by $cdef$

$$\text{i.e., } \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$$

$$\text{and } \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$$

On adding, we get

$$\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$$\text{or } af(b + e) = be(a + f)$$

Hence, $(a, b) R(e, f)$

$$\therefore R \text{ is transitive.} \quad \dots(vi)$$

From equations (i), (iii) and (vi), R is an equivalence relation.



OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

- Q. 1. Let set $X = \{1, 2, 3\}$ and a relation R is defined in X as: $R = \{(1, 3), (2, 2), (3, 2)\}$, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are
- (A) $\{(1, 1), (2, 3), (1, 2)\}$

- (B) $\{(3, 3), (3, 1), (1, 2)\}$
 (C) $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$
 (D) $\{(1, 1), (3, 3), (3, 1), (1, 2)\}$

[CBSE TERM-I 2021-22]

Ans. Option (C) is correct.

Explanation:

- (i) R is reflexive if it contains $\{(1, 1), (2, 2) \text{ and } (3, 3)\}$.

Since, $(2, 2) \in R$. So, we need to add $(1, 1)$ and $(2, 2)$ to make R reflexive.

- (ii) R is symmetric if it contains $\{(2, 2), (1, 3), (3, 1), (3, 2), (2, 3)\}$.

Since, $\{(2, 2), (1, 3), (3, 2)\} \in R$. So, we need to add $(3, 1)$ and $(2, 3)$.

Thus, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$.

Q. 2. If $R = \{(x, y); x, y \in \mathbb{Z}, x^2 + y^2 \leq 4\}$ is a relation in set $\mathbb{Z}$, then domain of R is

- (A) $\{0, 1, 2\}$ (B) $\{-2, -1, 0, 1, 2\}$
(C) $\{0, -1, -2\}$ (D) $\{-1, 0, 1\}$

[CBSE TERM-I SQP 2021-22]

Ans. Option (B) is correct.

Explanation: Given, $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + y^2 \leq 4\}$

Let $y = 0$, then $x^2 \leq 4 \Rightarrow x = 0, \pm 1, \pm 2$

Thus, domain of $R = \{-2, -1, 0, 1, 2\}$

Q. 3. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A ?

- (A) $(1, 1)$ (B) $(1, 2)$
(C) $(2, 2)$ (D) $(3, 3)$

[CBSE TERM-I SQP 2021-22]

Ans. Option (B) is correct.

Q. 4. Let the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$. Then [1], the equivalence class containing 1, is :

- (A) $\{1, 5, 9\}$ (B) $\{0, 1, 2, 5\}$
(C) $\emptyset$ (D) A

[CBSE TERM-I SQP 2021-22]

Ans. Option (A) is correct.

Explanation: Equivalence class [1] is the set of elements related to 1 = $\{1, 5, 9\}$

Q. 5. Let R be the relation in the set $\mathbb{N}$ given by $R = \{(a, b) : a = b - 2, b > 6\}$, then :

- (A) $(2, 4) \in R$ (B) $(3, 8) \in R$
(C) $(6, 8) \in R$ (D) $(8, 7) \in R$

[CBSE TERM-I SQP 2021-22]

Ans. Option (C) is correct.

Explanation: $6 = 8 - 2$

$(6, 8)$ is an element of R .

Q. 6. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to $b \forall a, b \in T$. Then R is

- (A) reflexive but not transitive
(B) transitive but not symmetric
(C) equivalence relation
(D) None of these

Q. 7. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b . Then R is

- (A) symmetric but not transitive
(B) transitive but not symmetric
(C) neither symmetric nor transitive
(D) both symmetric and transitive

Q. 8. The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are

- (A) 1 (B) 2
(C) 3 (D) 5

Ans. Option (D) is correct.

Explanation: Given that, $A = \{1, 2, 3\}$

Now, number of equivalence relations are as follows:

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

$$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$$

$$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$$

$$R_5 = \{(1, 2, 3) \Leftrightarrow A \times A = A^2\}$$

$\therefore$ Maximum number of equivalence relations on the set $A = \{1, 2, 3\} = 5$



SUBJECTIVE TYPE QUESTIONS



Very Short Answer Type Questions (1 mark each)

Q. 1. How many reflexive relations are possible in a set A whose $n(A) = 3$ [A] [CBSE SQP 2020-21]

Sol. 2^6 reflexive relations. 1
[CBSE Marking Scheme, 2020-21]

Detailed Answer:

Given, $n(A) = 3$

Total number of reflexive relations = $2^{n(n-1)}$
= $2^{3(3-1)} = 2^3 \times 2 = 2^6$



Commonly Made Error

- Since the reflexive relation should contain $(x, x) \in A$, mostly students write the answer as 3.



Answering Tip

- Number of reflexive relations on a set containing n elements is 2^{n^2-n} .

Q. 2. An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3 .

What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$.

[A] [CBSE SQP 2020-21]

Sol. $A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$ 1
[CBSE Marking Scheme 2020-21]

Q. 3. Let $A = \{1, 2, 3, 4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a, b)R(c, d)$ if $a + d = b + c$. Find the equivalence class $[(1, 3)]$. R&U [SQP 2017-18]

Q. 4. State the reason why the Relation $R = \{(a, b) : a \leq b^2\}$ on the set R of real numbers is not reflexive. R&U [NCERT SQP 2016-17]

Sol. $\frac{1}{2} > \left(\frac{1}{2}\right)^2 \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$.
Hence, R is not reflexive. 1
[CBSE Marking Scheme 2016]

Q. 5. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive. R [NCERT]

Sol. We know that, for a relation to be transitive,
 $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$.

Here, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.

$\therefore R$ is not transitive. 1

Q. 6. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N , write the range of R . R&U [O.D. Set I, II, III, 2014]

Sol. $R = \{(2, 3), (4, 2), (6, 1)\}$
Range = $\{3, 2, 1\}$ 1

Q. 7. Let $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$ be a relation. Find the range of R .

R&U [Foreign Set I, 2014]

Sol. Given $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$
 $\Rightarrow R = \{(2, 8), (3, 27)\}$ $\frac{1}{2}$
 $\therefore$ Range = $\{8, 27\}$ $\frac{1}{2}$

Q. 8. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$.

R&U [Delhi Comptt. Set I, II, III, 2014]

Sol. Given $R = \{(a, b) : 2 \text{ divides } (a - b)\}$
 $\forall a, b \in A = \{0, 1, 2, 3, 4, 5\}$
Equivalence class $[0] = \{0, 2, 4\}$ 1



Short Answer Type Questions-I (2 marks each)

Q. 1. Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Show that the relation R transitive? Write the equivalence class $[0]$.

A I R&U [CBSE SQP 2020-21]

Sol. Let 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$. 1

So 2 divides $[(a - b) + (b - c)]$

2 divides $(a - c)$: Yes relation R is transitive 1

$[0] = \{0, \pm 2, \pm 4, \pm 6, \dots\}$

[CBSE SQP Marking Scheme 2020]

Q. 2. Check if the relation R in the set R of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive.

A I R&U [Delhi Set I, II, III - 2020]

Q. 3. Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y) : y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive.

A I R&U [O. D Set I, II, III - 2020]

Sol. (i) As $(2, 4) \in R$ but $(4, 2) \notin R \Rightarrow R$ is not symmetric, 1

(ii) Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow b = \lambda a$ and $c = \mu b$

Now, $c = \mu b = \mu(\lambda a) \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive

1
[CBSE Marking Scheme 2020]

Detailed Answer:

$A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : y \text{ is divisible by } x\}$

(i) Symmetric

Let $(x, y) \in R$

y is divisible by x

$\therefore x$ is not necessarily divisible by y

$(y, x) \notin R$

e.g.,

$(1, 2) \in R$

2 is divisible by 1

but 1 is not divisible by 2

$(2, 1) \notin R$

Hence, Given Relation is not symmetric

(ii) Transitive

Let $(x, y) \in R$

y is divisible by x

...(i)

and

$(y, z) \in R$

z is divisible by y

...(ii)

From eq(i) and eq(ii)

z is divisible by x

$\therefore$

$(x, z) \in R$

e.g.,

$(1, 2) \in R$

2 is divisible by 1

...(i)

$(2, 4) \in R$

4 is divisible by 2

...(ii)

From eq(i) and eq(ii)

4 is divisible by 1

$(1, 4) \in R$

Hence, Given Relation is transitive.



Commonly Made Error

- Some students take the relation as "is a factor of" and go wrong.



Answering Tip

- 'is divisible by' should be taken as 'is a multiple of'

Q. 4. How many equivalence relations on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ are there in all ?

Justify your answer. [CBSE SQP 2016-17]

Sol. Equivalence relations could be the following :

$\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ and 1
 $\{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$ 1

So, only two equivalence relations.

[CBSE Marking Scheme 2016]

Q. 5. Let $f: X \rightarrow Y$ be a function. Define a relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$. Show that R is a transitive relation. [A]



Short Answer Type Questions-II (3 marks each)

Q. 1. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b \text{ is "divisible by } 2\}\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e. $[0]$.

[R&U] [CBSE SQP 2020-21]

Sol. (i) Reflexive:

Since, $a + a = 2a$ which is even

$\therefore (a, a) \in R \forall a \in Z$

Hence R is reflexive.

1/2

(ii) Symmetric:

If $(a, b) \in R$, then $a + b = 2\lambda$

$\Rightarrow b + a = 2\lambda$

$\Rightarrow (b, a) \in R$. Hence R is symmetric.

1

(iii) Transitive:

If $(a, b) \in R$ and $(b, c) \in R$

then $a + b = 2\lambda$... (i)

and $b + c = 2\mu$... (ii)

Adding (i) and (ii) we get

$$a + 2b + c = 2(\lambda + \mu)$$

$$\Rightarrow a + c = 2(\lambda + \mu - b)$$

$$\Rightarrow a + c = 2k$$

where $\lambda + \mu - b = k$

$\Rightarrow (a, c) \in R$

Hence R is transitive

$$[0] = \{\dots -4, -2, 0, 2, 4, \dots\} \quad 1\frac{1}{2}$$

[CBSE SQP Marking Scheme 2020]



Commonly Made Error

- Equivalence class of 0 is the set of all elements related to 0.



Answering Tip

- Mostly students go wrong in finding the equivalence class. Some students forget to write 0 in the equivalence class.

Q. 2. Prove that the relation R on Z , defined by $R = \{(x, y) : (x - y) \text{ is divisible by } 5\}$ is an equivalence relation. [A] [R&U] [CBSE O.D SET I - 2020]

Sol. For reflexive

$x - x = 0$, for every $x \in Z$ is divisible by 5 $\Rightarrow (x, x) \in R$

For symmetric 1/2

$(x, y) \in R \Rightarrow x - y$ is divisible by 5 $\Rightarrow y - x$ is divisible by 5 1/2

$\Rightarrow (y, x) \in R \Rightarrow R$ is symmetric

For transitive

Let $(x, y) \in R$ and $(y, z) \in R$

$(x, y) \in R \Rightarrow x - y = 5\lambda$... (i)

$(y, z) \in R \Rightarrow y - z = 5\mu$... (ii)

adding (i) and (ii), $x - z = 5(\lambda + \mu) = 5k$

$\Rightarrow (x, z) \in R \Rightarrow R$ is transitive

Hence R is an equivalence relation. 1

[CBSE Marking Scheme 2020 (modified)]

Q. 3. Show that the relation R on R defined as $R = \{(a, b) : a \leq b\}$, is reflexive, and transitive but not symmetric. [U] [CBSE Delhi Set III-2019]

Sol. Clearly $a \leq a \forall a \in R \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1/2

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R, a, b, c \in R$

$\Rightarrow a \leq b$ and $b \leq c \Rightarrow a \leq c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive.

1 1/2

For non-symmetric:

Let $a = 1, b = 2$, As $1 \leq 2 \Rightarrow (1, 2) \in R$

but $2 \not\leq 1 \Rightarrow (2, 1) \notin R$

$\Rightarrow R$ is non-symmetric.

1

[CBSE Marking Scheme 2019] (Modified)



Topper Answer, 2019

$R = \{(a, b) : a \leq b\}$

REFLEXIVE :
Every element $a \in \mathbb{R}$ is equal to itself
 $\Rightarrow a = a$
 $\Rightarrow a \leq a$ is true

$\therefore (a, a) \in R$ for all $a \in \mathbb{R}$ where $\mathbb{R}$ = set of real nos
 The relation is **REFLEXIVE** R is Relation.

TRANSITIVE :
For all $(a, b) \in R$ and $(b, c) \in R$
 $a \leq b$ and $b \leq c$ where $a, b, c \in \mathbb{R}$
 $\Rightarrow a \leq c$
 $\Rightarrow (a, c) \in R$
 $\therefore$ The set $\mathbb{R}$ relation is **TRANSITIVE**
 $\therefore$ for $(a, b), (b, c) \in R, (a, c) \in R$

SYMMETRIC : For relation to be symmetric,
 for all $(a, b) \in R, (b, a)$ should also exist in R
 $a \leq b$ ~~$a \leq b$~~ $a \leq b$
 $b \not\leq a \rightarrow$ This relation is true only $a = b = 1$.
 For eg: $\frac{1}{2} \leq 1 \Rightarrow (\frac{1}{2}, 1) \in R$
 but $1 \not\leq \frac{1}{2} \therefore (1, \frac{1}{2}) \notin R$
 $\therefore$ Relation is **NOT SYMMETRIC**



Commonly Made Error

- Some students use numerical examples to show that a reflexive, symmetric or transitive which is wrong.



Answering Tip

- Counter examples can be used only to show that a relation is not reflexive, symmetric or transitive.

Q. 4. Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3 is an equivalence relation.

R&U [CBSE Comptt. Set I, II, III 2018]

Sol. $(x - x) = 0$ is divisible by 3 for all $x \in Z$. So, $(x, x) \in R$
 $\therefore R$ is reflexive 1/2
 So $(x, y) \in R$ implies $(y, x) \in R, \forall x, y \in Z$
 $\Rightarrow R$ is symmetric, 1
 $(x - y)$ is divisible by 3 and $(y - z)$ is divisible by 3.
 So $(x - z) = (x - y) + (y - z)$ is divisible by 3. 1 1/2
 $\therefore (x, z) \in R \Rightarrow R$ is transitive
 Hence, R is an equivalence relation.

[CBSE Marking Scheme 2018 (modified)]

Q. 5. Check whether the relation R in the set R of real numbers, defined by $R = \{(a, b) : 1 + ab > 0\}$, is reflexive, symmetric or transitive.

 **R&U** [SQP 2018-19]

Q. 6. Show that the relation R in the set $N \times N$ defined by $(a, b) R (c, d)$ if $a^2 + d^2 = b^2 + c^2 \forall a, b, c, d \in N$, is an equivalence relation. **R&U** [SQP 2015-16]

Sol. Let $(a, b) \in N \times N$
 then,
 $\therefore a^2 + b^2 = a^2 + b^2$
 $\therefore (a, b) R (a, b)$
 Hence R is reflexive. 1/2
 Let $(a, b), (c, d) \in N \times N$ be such that
 $(a, b) R (c, d)$
 $\Rightarrow a^2 + d^2 = b^2 + c^2$
 $\Rightarrow c^2 + b^2 = d^2 + a^2$
 $\Rightarrow (c, d) R (a, b)$
 Hence, R is symmetric. 1
 Let $(a, b), (c, d), (e, f) \in N \times N$ be such that
 $(a, b) R (c, d), (c, d) R (e, f)$
 $\Rightarrow a^2 + d^2 = b^2 + c^2$... (i)
 and $c^2 + f^2 = d^2 + e^2$... (ii)
 Adding eqn. (i) and (ii),
 $\Rightarrow a^2 + d^2 + c^2 + f^2 = b^2 + c^2 + d^2 + e^2$
 $\Rightarrow a^2 + f^2 = b^2 + e^2$
 $\Rightarrow (a, b) R (e, f)$ 1 1/2
 Hence, R is transitive
 Since, R is reflexive, symmetric and transitive.
 Therefore, R is an equivalence relation.
[CBSE Marking Scheme 2015 (Modified)]



Commonly Made Error

- Students go wrong in solving problems involving ordered pairs.



Answering Tip

- Practice more problems involving relations with ordered pairs.



Long Answer Type Questions

(5 marks each)

Q. 1. Let $A = \{x \in Z : 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Write the equivalence class $[2]$.

A I R&U [CBSE Delhi & OD Set 2018]

Sol. Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall a \in A$ 1
 $\therefore (a, a) \in R, a \in A \therefore R$ is reflexive
Symmetric: let $(a, b) \in R$
 $\Rightarrow |a - b|$ is divisible by 4
 $\Rightarrow |b - a|$ is divisible by 4 ($\because |a - b| = |b - a|$) 1
 $\Rightarrow (b, a) \in R \therefore R$ is symmetric
Transitive: let $(a, b), (b, c) \in R$
 $\Rightarrow |a - b|$ & $|b - c|$ are divisible by 4
 $\Rightarrow a - b = \pm 4m, b - c = \pm 4n, m, n \in Z$
 Adding we get, $a - c = 4(\pm m \pm n)$ 2
 $\Rightarrow (a - c)$ is divisible by 4.
 $\Rightarrow |a - c|$ is divisible by 4 $\therefore (a, c) \in R$
 $\Rightarrow R$ is transitive
 Hence R is an equivalence relations in A
 Set of elements related to 1 is $(1, 5, 9)$, 1
 Equivalence class $[2] = \{2, 6, 10\}$

[CBSE Marking Scheme 2018 (modified)]

OR



Topper Answer, 2018

To show: R is an equivalence relation
Solution.
For Reflexive:-
 $(a, a) : a, a \in A$
 $a R a \quad \forall a \in A$
 $|a - a|$ is divisible by 4
 0 is divisible by 4
 which is true
 $a R b \quad \forall a, b \in A$

$|a-b|$ is divisible by 4

→ R is an symmetric relation since $(a,b) \in R$ also $(b,a) \in R$

for transitive relation :-

let $(a,b) \in R$ and $(b,c) \in R \quad \forall a,b,c \in A$

$a R b$ and $b R c \quad \forall a,b,c \in A$

$|a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

$$|a-b| = 4\lambda \quad \text{--- i)}$$

$$|b-c| = 4\mu \quad \text{--- ii)}$$

$$i) + ii)$$

$$|a-b+b-c| = 4(\lambda + \mu)$$

$$|a-c| = 4(\lambda + \mu)$$

$$\Rightarrow a R c \quad \forall a, c \in A$$

$$\Rightarrow (a,c) \in R$$

R is an transitive relation since $(a,b) \in R$, $(b,c) \in R$ and also $(a,c) \in R$

Hence R is reflexive, symmetric and transitive so it is a Equivalence relation.

The set of all elements related to A are :-

$$R = \{(1,5), (5,1), (1,9), (9,1), (1,1)\}$$

$$\text{Equivalence class } [2] = \{2, 6, 10\}$$

Q. 2. Show that the relation R in the Set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$ is an equivalence relation.

 **R&U** [O.D. Set I, II, III Comptt. 2015]

Topic-2

Functions

Concepts Covered • Types of functions and their identification



Revision Notes

- Domain:** If a function is expressed in the form $y = f(x)$, then domain of f means set of all those real values of x for which y is real (i.e., y is well-defined).

Remember the following points:

- Negative number should not occur under the square root (even root) i.e., expression under the square root sign must be always ≥ 0 .
 - Denominator should never be zero.
 - For $\log_b a$ to be defined, $a > 0$, $b > 0$ and $b \neq 1$. Also note that $\log_b 1$ is equal to zero i.e., 0.
- Range:** If a function is expressed in the form $y = f(x)$, then range of f means set of all possible real values of y corresponding to every value of x in its domain.

Remember the following points:

- At first find the domain of the given function.
 - If the domain does not contain an interval, then find the values of y putting these values of x from the domain. The set of all these values of y obtained will be the range.
 - If domain is the set of all real numbers R or set of all real numbers except a few points, then express x in terms of y and from this find the real values of y for which x is real and belongs to the domain.
- Function as a special type of relation:** A relation f from a set A to another set B is said to be a function (or mapping) from A to B if with every element (say x) of A , the relation f relates a unique element (say y) of B . This y is called f -image of x . Also x is called pre-image of y under f .

4. **Difference between relation and function:** A relation from a set A to another set B is any subset of $A \times B$; while a function f from A to B is a subset of $A \times B$ satisfying following conditions:

- (a) For every $x \in A$, there exists $y \in B$ such that $(x, y) \in f$.
 (b) If $(x, y) \in f$ and $(x, z) \in f$ then, $y = z$.

S. No.	Function	Relation
(i)	Each element of A must be related to some element of B .	There may be some elements of A which are not related to any element of B .

S. No.	Function	Relation
(ii)	An element of A should not be related to more than one element of B .	An element of A may be related to more than one element of B .

5. **Real valued function of a real variable:** If the domain and range of a function f are subsets of R (the set of real numbers), then f is said to be a **real valued function of a real variable** or a **real function**.

6. Some important real functions and their domain & range

S. No.	Function	Representation	Domain	Range
(i)	Identity function	$I(x) = x \forall x \in R$	R	R
(ii)	Modulus function or Absolute value function	$f(x) = x = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$	R	$[0, \infty)$
(iii)	Greatest integer function or Integral function or Step function	$f(x) = [x] \forall x \in R$	R	Z
(iv)	Smallest integer function	$f(x) = [x] \forall x \in R$	R	Z
(v)	Signum function	$f(x) = \begin{cases} x , & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ i.e., $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$	R	$\{-1, 0, 1\}$
(vi)	Exponential function	$f(x) = a^x, \forall a > 0, a \neq 1$	R	$(0, \infty)$
(vii)	Logarithmic function	$f(x) = \log_a x, \forall a \neq 1, a > 0 \text{ and } x > 0$	$(0, \infty)$	R

7. Types of Function

- (a) **One-one function (Injective function or Injection):** A function $f : A \rightarrow B$ is one-one function or injective function if distinct elements of A have distinct images in B .

Thus, $f : A \rightarrow B$ is one-one $\Leftrightarrow f(a) = f(b)$

$$\Rightarrow a = b, \forall a, b \in A$$

$$\Leftrightarrow a \neq b \Rightarrow f(a) \neq f(b) \forall a, b \in A.$$

- If A and B are two sets having m and n elements respectively such that $m \leq n$, then total number of one-one functions from set A to set B is ${}^nC_m \times m!$ i.e., nP_m .
- If $n(A) = n$, then the number of injective functions defined from A onto itself is $n!$.

ALGORITHM TO CHECK THE INJECTIVITY OF A FUNCTION

STEP 1: Take any two arbitrary elements a, b in the domain of f .

STEP 2: Put $f(a) = f(b)$.

STEP 3: Solve $f(a) = f(b)$. If it gives $a = b$ only, then f is a one-one function.

- (b) **Onto function (Surjective function or Surjection):** A function $f : A \rightarrow B$ is onto function or a surjective function if every element of B is the f -image of some element of A . That implies $f(A) = B$ or range of f is the co-domain of f .

Thus, $f : A \rightarrow B$ is onto $\Leftrightarrow f(A) = B$ i.e., range of f = co-domain of f .

ALGORITHM TO CHECK THE SURJECTIVITY OF A FUNCTION

STEP 1: Take an element $b \in B$, where B is the co-domain of the function.

STEP 2: Put $f(x) = b$.

STEP 3: Solve the equation $f(x) = b$ for x and obtain x in terms of b . Let $x = g(b)$.

STEP 4: If for all values of $b \in B$, the values of x obtained from $x = g(b)$ are in A , then f is onto. If there are some $b \in B$ for which values of x , given by $x = g(b)$, is not in A . Then f is not onto.



Mnemonics

Types of functions

Indian Syndicate Bank
Interpretations
Indian – injective
Syndicate – surjective
Bank – Bijective

Also note that a bijective function is also called a one-to-one function or one-to-one correspondence.

If $f: A \rightarrow B$ is a function such that,

(i) f is one-one $\Rightarrow n(A) \leq n(B)$.

(ii) f is onto $\Rightarrow n(B) \leq n(A)$.

For an ordinary finite set A , a one-one function $f: A \rightarrow A$ is necessarily onto and an onto function $f: A \rightarrow A$ is necessarily one-one for every finite set A .

(d) **Identity function:** The function $I_A: A \rightarrow A$; $I_A(x) = x, \forall x \in A$ is called an identity function on A .

Note:

- Domain (I_A) = A and Range (I_A) = A .

(e) **Equal function:** Two functions f and g having the same domain D are said to be equal if $f(x) = g(x)$ for all $x \in D$.

8. Constant and Types of Variables

(a) **Constant:** A constant is a symbol which retains the same value throughout a set of operations. So, a symbol which denotes a particular number is a constant. Constants are usually denoted by the symbols $a, b, c, k, l, m, \dots$ etc.

(b) **Variable:** It is a symbol which takes a number of values *i.e.*, it can take any arbitrary values over the interval on which it has been defined. For example, if x is a variable over R (set of real numbers) then we mean that x can denote any arbitrary real number. Variables are usually denoted by the symbols $x, y, z, u, v, \dots$ etc.

(i) **Independent variable:** The variable which can take an arbitrary value from a given set is termed as an independent variable.

(ii) **Dependent variable:** The variable whose value depends on the independent variable is called a dependent variable.

9. Defining a Function

Consider A and B be two non-empty sets, then a rule f which associates **each element of A with a unique element of B** is called a function or the mapping from A to B or f maps A to B . If f is a mapping from A to B , then we write $f: A \rightarrow B$ which is read as ' f is mapping from A to B ' or ' f is a function from A to B '.

If f associates $a \in A$ to $b \in B$, then we say that ' b is the **image of the element a under the function f** ' or ' b is the **f -image of a** ' or '**the value of f at a** ' and denotes it by $f(a)$ and we write $b = f(a)$. The element a is called the **pre-image or inverse-image of b** .

Thus for a bijective function from A to B ,

- A and B should be non-empty.
- Each element of A should have image in B .
- No element of A should have more than one image in B .
- If A and B have respectively m and n number of elements then the **number of functions defined from A to B is n^m** .

10. Domain, Co-domain and Range of a function

The set A is called the **domain** of the function f and the set B is called the **co-domain**. The set of the images of all the elements of A under the function f is called the **range of the function f** and is denoted as $f(A)$.

Thus range of the function f is $f(A) = \{f(x) : x \in A\}$.

Clearly $f(A) = B$ for a bijective function.

Note:

- It is necessary that every f -image is in B ; but there may be some elements in B which are not the f -images of any element of A *i.e.*, whose pre-image under f is not in A .
- Two or more elements of A may have same image in B .
- $f: x \rightarrow y$ means that under the function f from A to B , an element x of A has image y in B .
- Usually we denote the function f by writing $y = f(x)$ and read it as ' **y is a function of x** '.

Example 1

Determine whether the function $f: A \rightarrow B$ defined by $f(x) = 4x + 7, x \in A$ is one-one.

Show that no two elements in domain have same image in codomain.

Solution :

Given, $f: A \rightarrow B$ defined by $f(x) = 4x + 7, x \in A$

Let, $x_1, x_2 \in A$, such that $f(x_1) = f(x_2)$

$$\Rightarrow 4x_1 + 7 = 4x_2 + 7 \Rightarrow 4x_1 = 4x_2 \Rightarrow x_1 = x_2$$

So, f is one-one function.



OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

- Q. 1. Let $X = \{x^2 : x \in \mathbb{N}\}$ and the relation $f : \mathbb{N} \rightarrow X$ is defined by $f(x) = x^2, x \in \mathbb{N}$. Then, this function is
 (A) injective only (B) not bijective
 (C) surjective only (D) bijective

[CBSE TERM-I 2021-22]

Ans. Option (A) is correct.

Explanation: Let $x_1, x_2 \in \mathbb{N}$
 $f(x_1) = f(x_2)$
 $\Rightarrow x_1^2 = x_2^2$
 $\Rightarrow x_1^2 - x_2^2 = 0$
 $\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$
 $\Rightarrow x_1 = x_2$
 $\{x_1 + x_1 \neq 0 \text{ as } x_1, x_2 \in \mathbb{N}\}$

Hence, $f(x)$ is injective.

Also, the elements like 2 and 3 have no pre-image in $\mathbb{N}$. Thus, $f(x)$ is not surjective.

- Q. 2. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2 + x^2$ is
 (A) not one-one
 (B) one-one
 (C) not onto
 (D) neither one-one nor onto

[CBSE TERM-I 2021-22]

Ans. Option (D) is correct.

Explanation: $f(x) = 2 + x^2$
 For one-one, $f(x_1) = f(x_2)$
 $\Rightarrow 2 + x_1^2 = 2 + x_2^2$
 $\Rightarrow x_1^2 = x_2^2$
 $\Rightarrow x_1 = \pm x_2$
 $\Rightarrow x_1 = x_2$
 or $x_1 = -x_2$

Thus, $f(x)$ is not one-one.

For onto

Let $f(x) = y$ such that $y \in \mathbb{R}$
 $\therefore x^2 = y - 2$
 $\Rightarrow x = \pm\sqrt{y-2}$

Put $y = -3$, we get

$\Rightarrow x = \pm\sqrt{-3-2} = \pm\sqrt{-5}$

- Q. 3. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2 + x^3$ is :
 (A) One-one but not onto
 (B) Not one-one but onto
 (C) Neither one-one nor onto
 (D) One-one and onto

[CBSE TERM-I SQP 2021-22]

Ans. Option (D) is correct.

Explanation: $f(x) = x^3$ is a bijective function.

- Q. 4. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B . Based on the given information, f is best defined as :

- (A) Surjective function (B) Injective function
 (C) Bijective function (D) None of these

[CBSE TERM-I SQP 2021-22]

Ans. Option (B) is correct.

Explanation: f is injective since every element in set B has at most one pre-image in set A .

- Q. 5. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is
 (A) 720 (B) 120
 (C) 0 (D) None of these

Ans. Option (C) is correct.

Explanation: We know that, if A and B are two non-empty finite sets containing m and n elements, respectively, then the number of one-one and onto mapping from A to B is

$n!$ if $m = n$

0, if $m \neq n$

Given that, $m = 5$ and $n = 6$

$\therefore m \neq n$

Number of one-one and onto mapping = 0

- Q. 6. Let $A = \{1, 2, 3, \dots, n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is
 (A) ${}^n P_2$ (B) $2^n - 2$
 (C) $2^n - 1$ (D) None of these

Ans. Option (B) is correct.

Explanation: Total number of functions from A to $B = 2^n$

Number of into functions = 2

Number of surjections from A to $B = 2^n - 2$

- Q. 7. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{1}{x}, \forall x \in \mathbb{R}$. Then

f is

- (A) one-one (B) onto
 (C) bijective (D) f is not defined

- Q. 8. Which of the following functions from $\mathbb{Z}$ into $\mathbb{Z}$ are bijections?

- (A) $f(x) = x^3$ (B) $f(x) = x + 2$
 (C) $f(x) = 2x + 1$ (D) $f(x) = x^2 + 1$

Ans. Option (B) is correct.

Explanation: For bijection on $\mathbb{Z}$, $f(x)$ must be one-one and onto.

Function $f(x) = x^2 + 1$ is many-one as $f(1) = f(-1)$

Range of $f(x) = x^3$ is not $\mathbb{Z}$ for $x \in \mathbb{Z}$.

Also $f(x) = 2x + 1$ takes only values of type $= 2k + 1$ for $x \in k \in \mathbb{Z}$

But $f(x) = x + 2$ takes all integral values for $x \in \mathbb{Z}$

Hence $f(x) = x + 2$ is bijection of $\mathbb{Z}$.

- Q. 9. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x^4$. Choose the correct answer.

- (A) f is one-one onto
 (B) f is many-one onto
 (C) f is one-one but not onto
 (D) f is neither one-one nor onto

Ans. Option (D) is correct.

Explanation: We know that $f: R \rightarrow R$ is defined as $f(x) = x^4$.

Let $x, y \in R$ such that $f(x) = f(y)$

$$\Rightarrow x^4 = y^4$$

$$\Rightarrow x = \pm y$$

$$\therefore f(x) = f(y)$$

does not imply that $x = y$.

For example, $f(1) = f(-1) = 1$

$\therefore f$ is not one-one.

Consider an element 2 in co-domain R . It is clear that there does not exist any x in domain R such that $f(x) = 2$.

$\therefore f$ is not onto.

Hence, function f is neither one-one nor onto.

Q. 10. Let $f: R \rightarrow R$ be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto



SUBJECTIVE TYPE QUESTIONS



Very Short Answer Type Questions (1 mark each)

Q. 1. Check whether the function $f: R \rightarrow R$ defined as $f(x) = x^3$ is one-one or not.



Q. 2. A relation R in the set of real numbers R defined as $R = \{(a, b) : \sqrt{a} = b\}$ is a function or not. Justify

R&U [CBSE SQP - 2021]

Sol. Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$ 1
 $\therefore \sqrt{a} = b$ is not a function
[CBSE SQP Marking Scheme 2021]

Q. 3. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B . State whether f is one-one or not.



[CBSE SQP 2020-21]



Short Answer Type Questions-I (2 marks each)

Q. 1. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined

as $f(x) = \frac{4x+3}{6x-4}$ is one-one. A

Q. 2. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined

as $f(x) = \frac{4x+3}{6x-4}$ is onto. A

Q. 3. Show that the function $f: R \rightarrow R$ defined as $f(x) = x^2$ is neither one-one nor onto.

Sol. Given, a function $f: R \rightarrow R$ defined as $f(x) = x^2$

For one-one Here, at $x = 1, f(1) = 1$

and at $x = -1, f(-1) = (-1)^2 = 1$

Thus, $f(1) = f(-1) = 1$, but $1 \neq -1$

So, f is not one-one.

For onto Let $y \in R$ (codomain) be any arbitrary element.

$$\text{Then, } y = f(x)$$

$$\Rightarrow y = x^2$$

$$\Rightarrow x = \pm\sqrt{y}$$

$$\Rightarrow x = \pm\sqrt{y}$$

Now, for $y = -2 \in R, x \notin R$

So, f is not onto.

Hence, given function is neither one-one nor onto.

Q. 4. Show that the function $f: N \rightarrow N$, given by $f(x) = 2x$ is one-one but not onto.

Sol. Given, a function $f: N \rightarrow N$, defined as $f(x) = 2x$

For one-one Let, $x_1, x_2 \in N$, such that $f(x_1) = f(x_2)$

$$\Rightarrow 2x_1 = 2x_2$$

$$\Rightarrow x_1 = x_2$$

So, f is one-one

For onto Let $y \in N$ (codomain) be any arbitrary element.

$$\text{Then, } y = f(x)$$

$$\Rightarrow y = 2x$$

$$\Rightarrow x = \frac{y}{2}$$

Now, for $y = 1, x = \frac{1}{2} \notin N$.

Thus, $y = 1 \in N$ (codomain) does not have a pre-image in domain (N). So, f is not onto.



Short Answer Type Questions-II (3 marks each)

Q. 1. Show that the function $f: R \rightarrow R$ defined by $f(x) =$

$$\frac{x}{x^2+1}, \forall x \in R \text{ is neither one-one nor onto.}$$

Sol. Checking for one-one:

$$\text{here } f(x) = f\left(\frac{1}{x}\right). \text{ For example } f(2) = f\left(\frac{1}{2}\right)$$

$\therefore f$ is not one-one 1½
 Checking for onto:
 Let $y = 1 \in R$ (co-domain). Then
 $y = f(x) \Rightarrow \frac{x}{x^2 + 1} = 1$
 $\Rightarrow x^2 - x + 1 = 0$, which has no real roots.
 $\therefore R_f \neq \text{co-domain} \Rightarrow f$ is not onto. 1½
[CBSE SQP Marking Scheme 2020 (Modified)]

Q. 2. Show that the function $f: N \rightarrow N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$ for every $x > 2$, is onto but not one-one.

Sol. We have a function $f: N \rightarrow N$, defined as
 $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$.
For one-one Since $f(1) = f(2) = 1$, therefore 1 and have same image, namely 1. So, f is not one-one.
For onto Note that $y = 1$ has two pre-images, namely 1 and 2. Now, let $y \in N, y \neq 1$ be any arbitrary element.
 Then, $y = f(x) \Rightarrow y = x - 1$
 $\Rightarrow x = y + 1 > 2$ for every $y \in N, y \neq 1$.
 Thus, for every $y \in N, y \neq 1$, there exists $x = y + 1$ such that
 $f(x) = f(y + 1) = y + 1 - 1 = y$
 Hence, f is onto.

Q. 3. Prove that the function $f: N \rightarrow N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

[CBSE Delhi Set III-2019]

Sol. For one-one. Let $x_1, x_2 \in N$.
 $f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$
 $\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$
 $\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$
 $(\because x_1, x_2 \in N)$ 1½
 $\Rightarrow f$ is one-one.

For not onto.
 for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ 1½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $f(x) = x^2 + x + 1$
 for $x_1, x_2 \in N$
 $f(x_1) = f(x_2)$
 $x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$
 $x_1^2 - x_2^2 + x_1 - x_2 = 0$
 $(x_1 - x_2)(x_1 + x_2) + (x_1 - x_2) = 0$
 $(x_1 - x_2)(x_1 + x_2 + 1) = 0$

Therefore, the given function is one-one.

Also, f is not onto as for $1 \in N$, there does exist any 'x' in $f(x) = 1$.



Long Answer Type Questions (5 marks each)

Q. 1. Check which of the following function is onto or into.

- (i) $f: A \rightarrow B$, given by $f(x) = 3x$, where $A = \{0, 1, 2\}$ and $B = \{0, 3, 6\}$.
 (ii) $f: Z \rightarrow Z$, given by $f(x) = 3x + 2$, where $Z = \text{set of integers}$.

Sol. (i) We have a function $f: A \rightarrow B$, given by $f(x) = 3x$, where $A = \{0, 1, 2\}$ and $B = \{0, 3, 6\}$
 Let $y \in B$ be any arbitrary element.

Then, $y = f(x) \Rightarrow y = 3x \Rightarrow x = \frac{y}{3}$

Now, at $y = 0, x = \frac{0}{3} = 0 \in A$

At $y = 3, x = \frac{3}{3} = 1 \in A$

At $y = 6, x = \frac{6}{3} = 2 \in A$

Thus, for each element y of B , there is a pre-image in A .

(ii) We have a function $f: Z \rightarrow Z$, given by $f(x) = 3x + 2$.
 Let $y \in Z$, (codomain of f) be any arbitrary element.

Q. 2. Let R be the set of all non-zero real number. Then, show that $f: R \rightarrow R$, given by $f(x) = \frac{1}{x}$ is one-one and onto.

Sol. Given, $f(x) = \frac{1}{x}$

For one-one Let $x_1, x_2 \in R$, such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$ [put x_1 and x_2 in $f(x) = \frac{1}{x}$]

$\Rightarrow x_1 = x_2$

So, f is one-one

For onto Let $y \in R$ be any arbitrary element.

Then, $y = f(x)$

$\Rightarrow y = \frac{1}{x}$

$\Rightarrow x = \frac{1}{y}$ [expressing x in terms of y]

It is clear that for every $y \in R$ (codomain), $x \in R$ (domain)

Thus, for each $y \in R$ (codomain), there exist

$x = \frac{1}{y} \in R$ (domain), such that $f(x) = f\left(\frac{1}{y}\right) = \frac{1}{\left(\frac{1}{y}\right)} = y$

[i.e., every element of codomain has pre-image in domain]

So, f is onto.



COMPETENCY BASED QUESTIONS



Case based MCQs

Attempt any four sub-parts from each question.
Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation ' R ' is defined on I as follows:

ONE – NATION
ONE – ELECTION
FESTIVAL OF DEMOCRACY
GENERAL ELECTION - 2019



$R = \{(V_1, V_2) : V_1, V_2 \in I \text{ and both use their voting right in general election - 2019}\}$

[CBSE QB 2021]

Q. 1. Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election - 2019. Which of the following is true?

- (A) $(X, Y) \in R$
- (B) $(Y, X) \in R$
- (C) $(X, X) \notin R$
- (D) $(X, Y) \notin R$

Ans. Option (D) is correct.

Explanation: $(X, Y) \notin R$.

$\therefore X$ exercised his voting right while, Y did not cast her vote in general election-2019

And $R = \{(V_1, V_2) : V_1, V_2 \in I \text{ and both use their voting right in general election-2019}\}$

Q. 2. Mr. 'X' and his wife 'W' both exercised their voting right in general election -2019, Which of the following is true?

- (A) both (X, W) and $(W, X) \in R$
- (B) $(X, W) \in R$ but $(W, X) \notin R$
- (C) both (X, W) and $(W, X) \notin R$
- (D) $(W, X) \in R$ but $(X, W) \notin R$

Ans. Option (A) is correct.

Q. 3. Three friends F_1, F_2 and F_3 exercised their voting right in general election-2019, then which of the following is true?

- (A) $(F_1, F_2) \in R, (F_2, F_3) \in R$ and $(F_1, F_3) \in R$
- (B) $(F_1, F_2) \in R, (F_2, F_3) \in R$ and $(F_1, F_3) \notin R$
- (C) $(F_1, F_2) \in R, (F_2, F_3) \in R$ but $(F_1, F_3) \notin R$
- (D) $(F_1, F_2) \notin R, (F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

Ans. Option (A) is correct.

Q. 4. The above defined relation R is _____

- (A) Symmetric and transitive but not reflexive
- (B) Universal relation
- (C) Equivalence relation
- (D) Reflexive but not symmetric and transitive

Ans. Option (C) is correct.

Explanation: R is reflexive, since every person is friend or itself.

i.e., $(F_1, F_2) \in R$

Further, $(F_1, F_2) \in R$

$\Rightarrow F_1$ is friend of F_2

$\Rightarrow F_2$ is friend of F_1

$\Rightarrow (F_2, F_1) \in R$

$\Rightarrow R$ is symmetric

Moreover, $(F_1, F_2), (F_2, F_3) \in R$

$\Rightarrow F_1$ is friend of F_2 and F_2 is friend of F_3 .

$\Rightarrow F_1$ is a friend of F_3 .

$\Rightarrow (F_1, F_3) \in R$

Therefore, R is an equivalence relation.

Q. 5. Mr. Shyam exercised his voting right in General Election - 2019, then Mr. Shyam is related to which of the following?

- (A) All those eligible voters who cast their votes
- (B) Family members of Mr. Shyam
- (C) All citizens of India
- (D) Eligible voters of India

Ans. Option (A) is correct.

II. Read the following text and answer the following questions on the basis of the same:

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1, 2, 3, 4, 5, 6\}$. Let A be the set of players while B be the set of all possible outcomes.



$A = \{S, D\}, B = \{1, 2, 3, 4, 5, 6\}$ [CBSE QB 2021]

Q. 1. Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : y \text{ is divisible by } x\}$ is

- (A) Reflexive and transitive but not symmetric
- (B) Reflexive and symmetric but not transitive
- (C) Not reflexive but symmetric and transitive
- (D) Equivalence

Ans. Option (A) is correct.

Explanation: R is reflexive, since every element of B i.e.,

$B = \{1, 2, 3, 4, 5, 6\}$ is divisible by itself.

i.e., $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \in R$

further, $(1, 2) \in R$

but $(2, 1) \notin R$

Moreover,

$(1, 2), (2, 4) \in R$

$\Rightarrow (1, 4) \in R$

$\Rightarrow R$ is transitive.

Therefore, R is reflexive and transitive but not symmetric.

Q. 2. Raji wants to know the number of functions from A to B . How many number of functions are possible?

- (A) 6^2
- (B) 2^6
- (C) $6!$
- (D) 2^{12}

Ans. Option (A) is correct.

Q. 3. Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then R is

- (A) Symmetric
- (B) Reflexive
- (C) Transitive
- (D) None of these

Ans. Option (D) is correct.

Explanation: $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$

R is not reflexive. $(3, 3) \notin R$

Since, $(1, 1), (3, 3), (4, 4), (6, 6) \in R$

R is not symmetric.

Because, for $(1, 2) \in R$ there

$(2, 1) \notin R$.

R is not transitive.

Because for all element of B there does not exist,

$(a, b) (b, c) \in R$ and $(a, c) \in R$.

Q. 4. Raji wants to know the number of relations possible from A to B . How many numbers of relations are possible?

- (A) 6^2
- (B) 2^6
- (C) $6!$
- (D) 2^{12}

Ans. Option (D) is correct.

Q. 5. Let $R : B \rightarrow B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$, then R is

- (A) Symmetric
- (B) Reflexive and Transitive
- (C) Transitive and symmetric
- (D) Equivalence

Ans. Option (B) is correct.

III. Read the following text and answer the following questions on the basis of the same:

An organization conducted bike race under 2 different categories—boys and girls. Totally there

were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ $G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. [CBSE QB 2021]



Ravi decides to explore these sets for various types of relations and functions

Q. 1. Ravi wishes to form all the relations possible from B to G . How many such relations are possible?

- (A) 2^6
- (B) 2^5
- (C) 0
- (D) 2^3

Ans. Option (A) is correct.

Q. 2. Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of same sex}\}$, Then this relation R is _____

- (A) Equivalence
- (B) Reflexive only
- (C) Reflexive and symmetric but not transitive
- (D) Reflexive and transitive but not symmetric

Ans. Option (A) is correct.

Explanation:

$R : B \rightarrow B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of same sex}\}$

R is reflexive, since, $(x, x) \in R$

R is symmetric, since, $(x, y) \in R$ and $(y, x) \in R$

R is transitive. For $a, b, c \in B$

$\exists (a, b) (b, c) \in R$

and

$(a, c) \in R$.

Therefore R is equivalence relation.

Q. 3. Ravi wants to know among those relations, how many functions can be formed from B to G ?

- (A) 2^2
- (B) 2^{12}
- (C) 3^2
- (D) 2^3

Ans. Option (D) is correct.

Q. 4. Let $R : B \rightarrow G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$, then R is _____

- (A) Injective
- (B) Surjective
- (C) Neither Surjective nor Injective
- (D) Surjective and Injective

Ans. Option (B) is correct.

Explanation:

$R : B \rightarrow G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$
 R is surjective, since, every element of G is the image of some element of B under R , i.e., For $g_1, g_2 \in G$, there exists an elements $b_1, b_2, b_3 \in B$,
 $(b_1, g_1), (b_2, g_2), (b_3, g_1) \in R$.

Q. 5. Ravi wants to find the number of injective functions from B to G . How many numbers of injective functions are possible?

- (A) 0 (B) 2!
 (C) 3! (D) 0!

Ans. Option (A) is correct.

IV. Read the following text and answer the following questions on the basis of the same:

Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L .

[CBSE QB 2021]



Q. 1. Let relation R be defined by $R = \{(L_1, L_2) : L_1 \parallel L_2 \text{ where } L_1, L_2 \in L\}$ then R is _____ relation

- (A) Equivalence
 (B) Only reflexive
 (C) Not reflexive
 (D) Symmetric but not transitive

Ans. Option (A) is correct.

Explanation: Let relation R be defined by
 $R = \{(L_1, L_2) : L_1 \parallel L_2 \text{ where } L_1, L_2 \in L\}$.
 R is reflexive, since every line is parallel to itself.
 Further, $(L_2, L_1) \in R$
 $\Rightarrow L_1$ is parallel to L_2
 $\Rightarrow L_2$ is parallel to L_1
 $\Rightarrow (L_2, L_1) \in R$
 Hence, R is symmetric.
 Moreover, $(L_1, L_2), (L_2, L_3) \in R$
 $\Rightarrow L_1$ is parallel to L_2 and L_2 is parallel to L_3
 $\Rightarrow L_1$ is parallel to L_3
 $\Rightarrow (L_1, L_3) \in R$
 Therefore, R is an equivalence relation

Q. 2. Let $R = \{(L_1, L_2) : L_1 \perp L_2 \text{ where } L_1, L_2 \in L\}$ which of the following is true?

- (A) R is Symmetric but neither reflexive nor transitive
 (B) R is Reflexive and transitive but not symmetric
 (C) R is Reflexive but neither symmetric nor transitive
 (D) R is an Equivalence relation

Ans. Option (A) is correct.

Explanation: R is not reflexive, as a line L_1 can not be perpendicular to itself, i.e., $(L_1, L_1) \notin R$.
 R is symmetric as $(L_1, L_2) \in R$
 As, L_1 is perpendicular to L_2
 and L_2 is perpendicular to L_1
 $(L_2, L_1) \in R$
 R is not transitive. Indeed, it L_1 is perpendicular to L_2 and L_2 is perpendicular to L_3 , then L_1 can never be perpendicular to L_3 .
 In fact L_1 is parallel to L_3 ,
 i.e., $(L_1, L_2) \in R, (L_2, L_3) \in R$ but $(L_1, L_3) \notin R$
 i.e., symmetric but neither reflexive nor transitive.

Q. 3. The function $f : R \rightarrow R$ defined by $f(x) = x - 4$ is _____

- (A) Bijective
 (B) Surjective but not injective
 (C) Injective but not Surjective
 (D) Neither Surjective nor Injective

Ans. Option (A) is correct.

Explanation:
 The function f is one-one,
 for $f(x_1) = f(x_2)$
 $\Rightarrow x_1 - 4 = x_2 - 4$
 $\Rightarrow x_1 = x_2$
 Also, given any real number y in R , there exists $y + 4$ in R
 Such that $f(y + 4) = y + 4 - 4 = y$
 Hence, f is onto
 Hence, function is both one-one and onto, i.e., bijective.

Q. 4. Let $f : R \rightarrow R$ be defined by $f(x) = x - 4$. Then the range of $f(x)$ is _____

- (A) R (B) Z
 (C) W (D) Q

Ans. Option (A) is correct.

Explanation: Range of $f(x)$ is R

Q. 5. Let $R = \{(L_1, L_2) : L_1 \parallel L_2 \text{ and } L_1 : y = x - 4\}$ then which of the following can be taken as L_2 ?

- (A) $2x - 2y + 5 = 0$ (B) $2x + y = 5$
 (C) $2x + 2y + 7 = 0$ (D) $x + y = 7$

Ans. Option (A) is correct.

Explanation: Since, $L_1 \parallel L_2$
 then slope of both the lines should be same.
 Slope of $L_1 = 1$
 $\Rightarrow$ Slope of $L_2 = 1$
 And $2x - 2y + 5 = 0$

$$-2y = -2x - 5$$

$$y = x + \frac{5}{2}$$

Slope of $2x - 2y + 5 = 0$ is 1

So, $2x - 2y + 5 = 0$ can be taken as L_2 .



Case based Subjective Questions (4 mark each)

(Each Sub-part carries 2 marks)

- I. Read the following text and answer the following questions on the basis of the same:

Rohan is confused in the Mathematics topic 'Relation and equivalence relation'. To clear his concepts on the topic, he took help his elder brother. He has following notes on this topic.

Relation : A relation R from a set A to a set B is a subset of the cartesian product $A \times B$ obtained by describing a relationship between first element x and the second element ' y ' of the ordered pairs in $A \times B$. A relation R in a set A is called. :

Reflexive : If $(a, a) \in R \forall a \in A$.

Symmetric : If $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R \forall a_1, a_2 \in R$.

Transitive : If $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R \forall a_1, a_2, a_3 \in A$

Equivalence Relation : A relation R in a set A is an equivalence relation if R is reflexive, symmetric and transitive.

- Q. 1. Show that relation defined by $R_1 = \{(x, y) \mid x^2 = y^2\}$ $x, y \in \mathbb{R}$ is an equivalence relation.

Sol. Given relation $R_1 = \{(x, y) \mid x^2 = y^2\}$

Reflexive : For all $x \in \mathbb{R}$, $x^2 = x^2$, so, $(x, x) \in R_1$

Hence, R_1 is reflexive relation.

Symmetric : For all $x, y \in \mathbb{R}$

If $x^2 = y^2$ then $y^2 = x^2$

Hence, R_1 is symmetric relation.

Transitive : For all $x, y \in \mathbb{R}$, $x^2 = y^2$ and for all $y, z \in \mathbb{R}$ $y^2 = z^2$

$\therefore x^2 = y^2 = z^2$, for all $x, y, z \in \mathbb{R}$

Hence, R_1 is transitive.

Thus, R_1 is an equivalence relation.

- Q. 2. Check whether the relation (R) ' x greater than y ' for all $x, y \in \mathbb{N}$ is reflexive, symmetric or transitive.

Sol. Given, x greater than y , $\forall x, y \in \mathbb{N}$

$\Rightarrow x > y \forall x, y \in \mathbb{N}$

Reflexive : Now, for $(x, x) \in R$

Therefore, $x > x$ is not true for any $x \in \mathbb{N}$

Thus, R is not reflexive.

Symmetric : Now, let $(x, y) \in R$, then $x > y$

If $x > y$, then $y \not> x$ for any $x, y \in \mathbb{N}$

Thus, R is not symmetric.

Transitive : Now, let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow x > y$ and $y > z$

Therefore, $x > (z) \in R$ for all $x, y, z \in \mathbb{N}$

Thus, R is transitive.



Solutions for Practice Questions (Topic-1)

Multiple Choice Questions

6. Option (C) is correct.

Explanation: Consider that aRb , if a is congruent to b , $\forall a, b \in T$.

Then, $aRa \Rightarrow a \equiv a$,

Which is true for all $a \in T$

So, R is reflexive,

Let $aRb \Rightarrow a \equiv b$

$\Rightarrow b \equiv a$

$\Rightarrow bRa$

So, R is symmetric.

Let aRb and bRc

$\Rightarrow b \equiv b$ and $b \equiv a$

$\Rightarrow a \equiv c \Rightarrow aRc$

So, R is transitive

Hence, R is equivalence relation.

7. Option (B) is correct.

Explanation: $aRb \Rightarrow a$ is brother of b .

This does not mean b is also a brother of a as b can be a sister of a .

Hence, R is not symmetric.

$aRb \Rightarrow a$ is brother of b

and $bRc \Rightarrow b$ is a brother of c .

So, a is brother of c .

Hence, R is transitive.

Very Short Answer Type Questions

3. $[(1, 3)] = \{(x, y) \in A \times A : x + 3 = y + 1\}$
 $= \{(x, y) \in A \times A : y - x = 2\}$
 $= \{(1, 3), (2, 4)\}$

[CBSE Marking Scheme 2017-18]

Short Answer Type Questions-I

2. (i) $1, 2 \in \mathbb{R}$ such that $1 < 2 \Rightarrow (1, 2) \in R$, but since 2 is not less than 1 $\Rightarrow (2, 1) \notin R$. Hence R is not symmetric.

(ii) Let $(a, b) \in R$ and $(b, c) \in R$, $\therefore a < b$ and $b < c$

$\Rightarrow a < c \Rightarrow (a, c) \in R \therefore R$ is transitive.

[CBSE SQP Marking Scheme 2020]

Detailed Answer:

(i) It is not symmetric because if $a < b$ then $b < a$ is not true.

(ii) Here, if $a < b$ and $b < c$ then $a < c$ is also true for all $a, b, c \in \text{Real numbers}$. Therefore R is transitive.



Commonly Made Error

► Students use examples to show that the relation is transitive which is wrong.



Answering Tip

- Use only arbitrary elements to prove transitivity.

5. Let $(a, b), (b, c) \in R, f(a) = f(b), f(b) = f(c) \Rightarrow f(a) = f(c), (a, c) \in R$. Thus, Relation is transitive.

Short Answer Type Questions-II

5. Reflexive:

R is reflexive, as $1 + a.a = 1 + a^2 > 0 \Rightarrow (a, a) \in R$
 $\forall a \in R$ 1

Symmetric:

If $(a, b) \in R$
 then, $1 + ab > 0$
 $\Rightarrow 1 + ba > 0$
 $\Rightarrow (b, a) \in R$

Hence, R is symmetric. 1

Transitive:

Let $a = -8, b = -1, c = \frac{1}{2}$

Since, $1 + ab = 1 + (-8)(-1) = 9 > 0$
 $\therefore (a, b) \in R$

also, $1 + bc = 1 + (-1)\left(\frac{1}{2}\right) = \frac{1}{2} > 0$

$\therefore (b, c) \in R$

But, $1 + ac = 1 + (-8)\left(\frac{1}{2}\right) = -3 < 0$

Hence, R is not transitive. 1

[CBSE Marking Scheme 2018 (modified)]



Commonly Made Error

- Students use counter example to prove reflexive and symmetric.



Answering Tip

- Counter examples can be used only to show exceptions.

Long Answer Type Questions

2. Given $R = \{(a, b) : |a - b| \text{ is divisible by } 2\}$
 and $A = \{1, 2, 3, 4, 5\}$
 $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)$
 $(1, 3), (1, 5), (2, 4), (3, 5), (3, 1),$
 $(5, 1), (4, 2), (5, 3)\}$ 2

(i) $\forall a \in A, (a, a) \in R,$

$\therefore R$ is reflexive. 1

[As $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\} \in R$]

(ii) $\forall (a, b) \in A, (b, a) \in R,$

$\therefore R$ is symmetric. 1

[As $\{(1, 3), (1, 5), (2, 4), (3, 5), (3, 1), (5, 1), (4, 2), (5, 3)\} \in R$]

(iii) $\forall (a, b), (b, c) \in R, (a, c) \in R$

$\therefore R$ is transitive. 1

[As $\{(1, 3), (3, 1) \in R \Rightarrow (1, 1) \in R$ and similarly others]

$\therefore R$ is an equivalence relation.

[CBSE Marking Scheme 2015 (Modified)]



Solutions for Practice Questions (Topic-2)

Multiple Choice Questions

7. Option (D) is correct.

Explanation: We have, $f(x) = \frac{1}{x}, \forall x \in R$

For $x = 0, f(x)$ is not defined.

Hence, $f(x)$ is not a defined function.

10. Option (A) is correct.

Explanation: $f: R \rightarrow R$ is defined as $f(x) = 3x$.

Let $x, y \in R$ such that $f(x) = f(y)$

$$\Rightarrow 3x = 3y$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Also, for any real number (y) in co-domain R , there

exists $\frac{y}{3}$ in R such that $f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$.

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Very Short Answer Type Questions

1. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$ 1
 $\Rightarrow (x_1)^3 = (x_2)^3$
 $\Rightarrow x_1 = x_2$
 Hence $f(x)$ is one-one.

[CBSE SQP Marking Scheme 2021]



Commonly Made Error

- Students get confused between one-one and many-one functions.



Answering Tip

- Injectivity should be determined considering the domain and co-domain. A function which is one-one in a domain may not be one-one in another domain.

3. Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f: A \rightarrow B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$ i.e. $f(1) = 4, f(2) = 5$ and $f(3) = 6$.

It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one. 1

Short Answer Type Questions-I

1. Given $f(x) = \frac{4x+3}{6x-4}$
- Let, $f(x_1) = f(x_2)$,
- then $\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$ $\frac{1}{2}$
- or $(4x_1+3)(6x_2-4) = (6x_1-4)(4x_2+3)$ $\frac{1}{2}$
- or $24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 + 18x_1 - 16x_2 - 12$
- or $-16x_1 + 18x_2 = 18x_1 - 16x_2$ $\frac{1}{2}$
- or $-16x_1 - 18x_1 = -18x_2 - 16x_2$
- or $-34x_1 = -34x_2$
- or $x_1 = x_2$
- or f is one-one. $\frac{1}{2}$

2. Let, $y \in B = R - \left\{\frac{2}{3}\right\}$
- $\therefore y = f(x)$
- or $y = \frac{4x+3}{6x-4}$
- or $y(6x-4) = 4x+3$ $\frac{1}{2}$
- or $6xy - 4y = 4x+3$
- or $6xy - 4x = 4y+3$
- or $x(6y-4) = 4y+3$
- or $x = \frac{4y+3}{6y-4} \in B = R - \left\{\frac{2}{3}\right\}$ 1
- or For every value of y except $y = \frac{2}{3}$, there is a
- pre-image $x = \frac{4y+3}{6y-4} = g(y)$.
- or $x \in A$
- $\therefore f$ is onto. $\frac{1}{2}$



REFLECTIONS

- In this chapter we have covered the different types of relations and functions. Look around you and pick some real life relations say 'is the father of', 'is the friend of' etc and check whether they are reflexive, symmetric and transitive.

